

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE  
THE NATIONAL TECHNICAL UNIVERSITY OF UKRAINE  
“Igor Sikorsky Kyiv Polytechnic Institute”  
FACULTY OF PHYSICS AND MATHEMATICS

APPROVED BY

Dean of the Faculty of  
Physics and Mathematics

\_\_\_\_\_ Volodymyr Vanin  
« \_\_\_\_ » \_\_\_\_\_ 2020

**HIGHER MATHEMATICS 1.**  
**Differential and integral calculus of a function of one variable.**

SYLLABUS OF THE CREDIT MODULE

The first bachelor level of higher education

Specialty 131 "Applied Mechanics"

Profile program Manufacturing Engineering

Full-time form of study

Approved by the methodical commission of

the Faculty of Physics and Mathematics

Protocol dated \_\_\_\_\_ 2020 № \_\_\_\_

Head of the methodical commission

\_\_\_\_\_ (Nadiia Reva)

« \_\_\_\_ » \_\_\_\_\_ 2020.

Syllabus of the Credit Module "Higher Mathematics 1. Differential and integral calculus of a function of one variable" is based on the Curriculum "Higher Mathematics".

DEVELOPER OF THE SYLLABUS OF THE CREDIT MODULE:

Associate Professor, PhD Ganna Zhuravska \_\_\_\_\_

The Syllabus of the Credit Module is approved at the meeting of the department of Mathematical Physics of the Faculty of Physics and Mathematics of "Igor Sikorsky Kyiv Polytechnic Institute"

Protocol dated \_\_\_\_\_2020 № \_\_\_\_

Head of the Department \_\_\_\_\_Volodymyr Gorbachuk

«\_\_\_\_\_» \_\_\_\_\_2020.

## 1. Description of the Credit Module

Level of HE, Specialty Profile Programs Form of Study	General Indicators	Credit Module's Characteristics
Level of HE <u>The first bachelor</u>	Name of the discipline to which the credit module belongs <u>Higher mathematics</u>	Lectures <u>18 hours</u>
Specialty <u>131 "Applied Mechanics"</u>	Cycle to which the credit module belongs <u>General training</u>	Practical Lessons <u>36 hours</u>
Profile Programs Manufacturing Engineering	The status of the discipline <u>Compulsory.</u>	Self-study work <u>81 hours</u> Including the implementation of individual tasks <u>10 hours</u>
	Semester <u>First</u>	Individual home work <u>Individual tasks</u>
Form of Study <u>Full-time</u>	Number of Credits (Hours) 4.5 (135)	Type and form of the semester's control: <u>Oral exam</u>

The credit module belongs to the discipline "Higher Mathematics" and is included in the cycle of natural-scientific trainings for bachelors of the specialty 131 "Applied Mechanics". This credit module has a close relationship with credit modules: "Higher Mathematics 2: Differential and Integral Calculus of the Function of Several Variables. Differential Equations" and "Higher Mathematics 3: Series. Theory of the Functions of a Complex Variable" of this discipline. Also it is used in other credit modules, which are included in the disciplines according to the curriculum for the specialty " Applied Mechanics ".

## 2. Learning Objectives and Tasks of Credit Module

The purpose of the credit module is to form students' abilities:

- acquiring the basic concepts and methods of the theory of limits, differential and integral calculus of the function of one variable;
- using methods of mathematics in engineering calculations;
- analyzing the results obtained;
- independently using and studying literature on mathematics.

According to the requirements of the syllabus students must demonstrate the following learning outcomes:

### **Knowledge:**

- basic definitions of the functions of one variable (domain, range, basic characteristics of functions, basic elementary functions and their graphs)
- the principles of differential calculus of functions of one variable (limit of a numerical sequence, limit of a function, concept of infinitesimals, concept of continuous function, the points of discontinuity of function, the tangent and the normal line to the curve, the derivative and the differential of a function, asymptotes, applications of derivatives: monotonicity, extrema and concavity, L'Hospital's rule)
- the bases of differential calculus of functions of many variables (partial derivatives, full differential, surfaces, extrema of the function of two variables);
- the bases of the integral calculus of functions of one and many variables (indefinite integral, properties of an indefinite integral, a table of integrals, methods of integration).

### **Skills:**

- to know of graphs of basic elementary functions, to perform transformations of graph, to find the roots of polynomials, factoring polynomials;
- to find the limits of sequences and functions, to compare infinitesimals;
- to investigate the function for continuity, to classify points of discontinuity and asymptotes of the function;
- to find derivatives and differentials of functions of one variable, to apply the derivative and differential calculus, to use of L'Hospital's rule;
- to make the investigation of functions;
- to find indefinite integrals by basic methods of integration.

### 3. The Structure of the Credit Module

Parts and Topics	Number of Hours			
	Total	Including		
		Lectures	Practical Training	Self-study
1	2	3	4	5
<b>Part 1. Theory of Limits.</b>				
Topic 1.1. The Concept of Sets. Binary Operations with Sets. The Set of Real Numbers and Their Properties. The Absolute Value of a Real Number.	1,5	0,5		1
Topic 1.2. The Concept of Function. Ways of Representing Functions. Properties of Functions. The Inverse Function. Composite Function. Basic Elementary Functions and Their Graphs.	3,5	0,5	2	1
Topic 1.3. Numerical Sequence. Basic Concepts. The Limit of a Numerical Sequence. An Infinitely Large Variable.	5	1	2	2
Topic 1.4. The Limit of a Function. One-sided Limits. Infinitely Large Functions. Infinitesimals and Their Properties.	5	1	2	2
Topic 1.5. Basic Theorems on Limits. The Limit of the Function $\frac{\sin x}{x}$ as $x \rightarrow 0$ . Number <b>e</b> . Natural Logarithms.	7	1	4	2
Topic 1.6. Comparing of Infinitesimals. Equivalent Infinitesimals and Their Applications.	5	1	2	2
Topic 1.7. Continuity of Functions. Classification the Points of Discontinuity of a Function. Basic Theorems on Continuous Functions.	5	1	2	2
Topic 1.8. Certain Properties of Continuous Functions.	1	1		
Test 1	4		2	2
<b>Total for Part 1</b>	<b>37</b>	<b>7</b>	<b>16</b>	<b>14</b>

**Part 2. Differential Calculus of a Function of one variable.**

Topic 2.1. Definition of Derivative. Mechanical, Physical and Geometric Meanings of the Derivative. The Equations of a Tangent and a Normal Lines. Differentiability of Functions. Basic Rules of Differentiation: Derivative of a Sum, a Product, and a Quotient. The Derivative of a Composite Function (Chain Rule). The Derivative of an Inverse Function.	5	1	1	3
Topic 2.2. Derivatives of Basic Elementary Functions: Power Function, Exponential Function, Logarithmic Function, Trigonometric Functions, Inverse Trigonometric Functions, Hyperbolic Functions. The Table of Derivatives.	4	1	1	2
Topic 2.3. The Derivative of a Function Represented Parametrically. The Derivative of an Implicit Function. The Logarithmic Differentiation.	5	1	2	2
Topic 2.4. Derivatives of Higher Orders. Mechanical Significance of the Second Derivative. Derivatives of Higher Orders of Implicit Functions and of Functions Represented Parametrically.	5	1	2	2
Topic 2.5. The Differential and Its Geometric Significance. Differentials of Higher Orders.	5	1	2	2
Topic 2.6. Basic Theorems of the Differential calculus. A Theorem on the Roots of a Derivative. A Theorem on Finite Increments. A Theorem on the Ratio of the Increments of Two Functions. The L'Hospital's Rule.	4	1	1	2
Topic 2.7. Taylor's and Maclaurin's Formulas. Maclaurin's formulas for Basic Elementary Functions.	4	1	1	2
Topic 2.8. The Monotonicity of a Function. Local Extrema of a	4	1	1	2

Function. The Largest and Smallest Values of a Function. Topic 2.9. Concavity of a Curve. Points of Inflection. Asymptotes. The General Plan for Investigating Functions and Constructing Graphs.	4	1	1	2
Test 2	4		2	2
<b>Total for Part 2</b>	<b>44</b>	<b>9</b>	<b>14</b>	<b>21</b>
<b>Part 3. Integral Calculus of Functions of one variable.</b>				
Topic 3.1. The Concept of an Antiderivative. The Indefinite Integral and its properties. Table of Integrals.	5	1	2	2
Topic 3.2. Integration by Substitution. Integration by Parts.	5	1	2	2
Test 3	4		2	2
<b>Total for Part 3</b>	<b>14</b>	<b>2</b>	<b>6</b>	<b>6</b>
Individual Tasks	10			10
Preparation to the Exam	30			30
<b>Total</b>	<b>135</b>	<b>18</b>	<b>36</b>	<b>81</b>

#### 4. Lectures

№	The title of the lecture and a list of the main subtopics (task for the self-study and readings)
1	<p>Introduction to the course.</p> <p>The Concept of Sets. Binary Operations with Sets. The Set of Real Numbers and Their Properties. The Absolute Value of a Real Number.</p> <p>The Concept of Function. Ways of Representing Functions. Properties of Functions. The Inverse Function. Composite Function. Basic Elementary Functions and Their Graphs.</p> <p>Task for the Self-study: learn the lecture.</p> <p>Readings: [1] Chapter I §1-9, [2] Chapter 1 §1.2, 1.1.</p>

2	<p>Numerical Sequence. Basic Concepts. The Limit of a Numerical Sequence. An Infinitely Large Variable.</p> <p>The Limit of a Function. One-sided Limits. Infinitely Large Functions. Infinitesimals and Their Properties.</p> <p>Task for the Self-study: learn the lecture.</p> <p>Readings: [1] Chapter II §1-4, [2] Chapter 9 §9.1, Chapter 5 §5.1.</p>
3	<p>Basic Theorems on Limits. The Limit of the Function <math>\frac{\sin x}{x}</math> as <math>x \rightarrow 0</math>.</p> <p>Number <b>e</b>. Natural Logarithms.</p> <p>Comparing of Infinitesimals. Equivalent Infinitesimals and Their Applications.</p> <p>Task for the Self-study: learn the lecture.</p> <p>Readings: [1] Chapter III §5-8, Chapter II §11, [2] Chapter 3 §3.3.</p>
4	<p>Continuity of Functions. Classification the Points of Discontinuity of a Function. Basic Theorems on Continuous Functions.</p> <p>Certain Properties of Continuous Functions.</p> <p>Task for the Self-study: to learn the lecture.</p> <p>Readings: [1] Chapter II §9, 10, [2] Chapter 3 §3.4.</p>
5	<p>Definition of Derivative. Mechanical, Physical and Geometric Meanings of the Derivative. The Equations of a Tangent and a Normal Lines.</p> <p>Differentiability of Functions. Basic Rules of Differentiation: Derivative of a Sum, a Product, and a Quotient. The Derivative of a Composite Function (Chain Rule). The Derivative of an Inverse Function.</p> <p>Derivatives of Basic Elementary Functions. The Table of Derivatives. The Derivative of a Function Represented Parametrically. The Derivative of an Implicit Function. The Logarithmic Differentiation.</p> <p>Task for the Self-study: learn the lecture.</p> <p>Readings: [1] Chapter III §1-12,14,15,18, [2] Chapter 2 §2.1-2.6, Chapter 7 §7.2, Chapter 8 §8.3.</p>
6	<p>Derivatives of Higher Orders. Mechanical Significance of the Second Derivative. Derivatives of Higher Orders of Implicit Functions and of Functions Represented Parametrically. The Differential and Its Geometric Significance. Differentials of Higher Orders.</p> <p>Basic Theorems of the Differential calculus. A Theorem on the Roots of a Derivative. A Theorem on Finite Increments. A Theorem on the Ratio of the Increments of Two Functions. The L'Hospital's Rule.</p> <p>Task for the Self-study: learn the lecture.</p> <p>Readings: [1] Chapter III §20-25, Chapter IV §1-5, [2] Chapter 2 §2.7, Chapter 5 §5.2.</p>



7	<p>Taylor's and Maclaurin's Formulas. Maclaurin's formulas for Basic Elementary Functions.</p> <p>The Monotonicity of a Function. Local Extrema of a Function. The Largest and Smallest Values of a Function.</p> <p>Task for the Self-study: learn the lecture.</p> <p>Readings: [1] Chapter IV §6,7, Chapter V §2-7, [2] Chapter 9 §9.10.</p>
8	<p>Concavity of a Curve. Points of Inflection. Asymptotes. The General Plan for Investigating Functions and Constructing Graphs.</p> <p>The Concept of an Antiderivative. The Indefinite Integral and its properties.</p> <p>Task for the Self-study: learn the lecture.</p> <p>Readings: [1] Chapter V §9-12, Chapter X §1-3, [2] Chapter 4 §4.1.</p>
9	<p>Table of Integrals. Examples.</p> <p>Integration by Substitution. Integration by Parts.</p> <p>Task for the Self-study: learn the lecture.</p> <p>Readings: [1] Chapter X §1-4,6, [2] Chapter 4 §4.1.</p>

### 5. Practical Trainings

N 3 /π	The title of the practical training and a list of the main subtopics (task for the self-study and readings)
1	<p>The Concept of Function. Ways of Representing Functions. Properties of Functions. The Inverse Function. Composite Function. Basic Elementary Functions and Their Graphs.</p> <p>Task for the Self-study: solve a homework assignment.</p>
2	<p>Numerical Sequence. Basic Concepts. The Limit of a Numerical Sequence. An Infinitely Large Variable.</p> <p>Task for the Self-study: solve a homework assignment.</p>
3	<p>The Limit of a Function. Infinitely Large Functions. Infinitesimals and Their Properties.</p> <p>Task for the Self-study: solve a homework assignment.</p>
4	<p>The Limit of the Function <math>\frac{\sin x}{x}</math> as <math>x \rightarrow 0</math>.</p> <p>Task for the Self-study: solve a homework assignment.</p>
5	<p>Number <b>e</b>. Natural Logarithms.</p> <p>Task for the Self-study: solve a homework assignment.</p>
6	<p>Comparing of Infinitesimals. Equivalent Infinitesimals and Their Applications.</p> <p>Task for the Self-study: solve a homework assignment.</p>
7	<p>Continuity of Functions. Classification the Points of Discontinuity of a Function.</p>

	Task for the Self-study: solve a homework assignment.
8	Test 1.
9	Derivative. The Table of Derivatives. Basic Rules of Differentiation: Derivative of a Sum, a Product, and a Quotient. Chain Rule. Task for the Self-study: solve a homework assignment.
10	The Derivative of a Function Represented Parametrically. The Derivative of an Implicit Function. The Logarithmic Differentiation. Task for the Self-study: solve a homework assignment.
11	Derivatives of Higher Orders. Derivatives of Higher Orders of Implicit Functions and of Functions Represented Parametrically. Task for the Self-study: solve a homework assignment.
12	Differentials. The Equations of a Tangent and a Normal Lines. Task for the Self-study: to solve a homework assignment.
13	Basic Theorems of the Differential calculus. The L'Hospital's Rule. Taylor's and Maclaurin's Formulas. Task for the Self-study: solve a homework assignment.
14	The Monotonicity and Local Extrema of a Function. The Largest and Smallest Values of a Function. Concavity of a Curve. Points of Inflection. Asymptotes. The General Plan for Investigating Functions and Constructing Graphs. Task for the Self-study: solve a homework assignment.
15	Test 2.
16	The Concept of an Antiderivative. The Indefinite Integral and its properties. Table of Integrals. Task for the Self-study: solve a homework assignment.
17	Integration by Substitution. Integration by Parts. Task for the Self-study: solve a homework assignment.
18	Test 3.

## 6. Individual tasks

It is planned one set of individual tasks to the topics of parts 1-3 in the first semester.

They are aimed at broadening and deepening theoretical knowledge of students and promoting independent solving of specific problems.

## **7. Tests**

According to the Syllabus of the discipline "Higher Mathematics" there is one modular test planned. This modular test consist of tree tests. Tests are designed to execute ongoing control and assessment of student's knowledge by the teacher.

Examples of tests are provided to the Curriculum of the Credit Module in the form of an appendix 1.

## **8. Assessment of learning outcomes**

The rating system is provided to the Curriculum of the Credit Module in the form of an appendix 2.

## **9. Learning Activities and Teaching Methods**

Basic teaching methods:

1. Solving examples and tasks during practical trainings through active and collective learning, part-search and research methods;
2. As an addition to traditional training activities there are proposed such activities as Internet mailing, Distance and online learning, online testing ect.

## **10. Readings**

1. N.Piscunov Differential and Integral Calculus/ N.Piscunov – Mir Publisher, Moscow, 1966 – 895 p.

2. H. Jerome Keisler Elementary Calculus: an Infinitesimal Approach/  
H. Jerome Keisler – On-line Edition. 2000

<https://www.math.wisc.edu/~keisler/calc.html>

## **10. Informational Support**

1. <http://tutorial.math.lamar.edu>
2. <https://www.khanacademy.org>
3. <https://www.vitutor.com>

## Appendix 1.

### Test 1.

1. Calculate the limits of sequences

$$1.1 \lim_{n \rightarrow \infty} \frac{4n^3 - 6n}{7n^5 + n^4 - 5n^3};$$

$$1.4 \lim_{x \rightarrow 0} \frac{\tan 13x}{\log_2(1 + 4x)};$$

$$1.7 \lim_{n \rightarrow \infty} \left( \frac{3n + 2}{3n + 1} \right)^{2-2n};$$

$$1.2 \lim_{x \rightarrow -5} \frac{x^2 - 25}{\arctan 2(x + 5)};$$

$$1.5 \lim_{x \rightarrow 0} \left( 1 + (e^{3x} - 1) \right)^{\frac{1}{\ln(1+2x)}};$$

$$1.8 \lim_{x \rightarrow 1} \frac{\sin 9\pi x}{\tan 7\pi x}.$$

$$1.3 \lim_{x \rightarrow 0} \frac{x^2 + 2x - 24x}{x^2 + x - 20x};$$

$$1.6 \lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x^2 + x - 6};$$

2. State where the discontinuities of function exist and what kind they are:

$$2.1 \ y = \frac{1}{3x - 9};$$

$$2.2 \ y = \begin{cases} 1, & -\infty < x < 1, \\ 1 - x, & 1 \leq x < 2, \\ \sqrt[3]{x-1}, & 2 \leq x < +\infty. \end{cases} \quad \text{Plot the graph of this function.}$$

### Test 2.

1. Find the first derivative: 1.1  $y^5 + x^2 5^y + \arctan e^x = 0$ ; 1.2  $y = (\sin x)^{\cot x}$ .

2. Find the second derivative:  $\begin{cases} x = \sinh 3t, \\ y = \tan 5t. \end{cases}$

3. Find the equation of tangent line to the curve  $y = (3x - 1)^2 + \sqrt[6]{x^5}$  at point  $x_0 = 1$ .

4. Find the *inclined* asymptote of a curve  $y = \frac{x^3 - 2x^2}{1 - x^2}$ .

5. Find for the function  $y = (x + 2)^3 x^2$

5.1 Intervals of increasing and decreasing;

5.2. Maxima and minima;

5.3 Intervals of concavity down and up. Inflection points.

### Test 3.

1. Find the indefinite integrals.

$$1.1 \int \cos^2 12x dx; \quad 1.2 \int \frac{2x}{\sqrt{1-(x^2)^2}} dx; \quad 1.3 \int \frac{\sqrt{\ln^3 x}}{x} dx; \quad 1.4 \int \cos^4 x \sin x dx;$$

$$1.5 \int 4^x \sin 4^x dx; \quad 1.6 \int (8-3x)e^{2x} dx; \quad 1.7 \int \frac{\ln^2 x}{x^5} dx$$

## Appendix 2.

### The Rating System

1. The student rating of the credit module is calculated from 100 points, 60 of them are the starting scores. Starting scores (during the semester) consists of the points that the student receives for:

- 1) responses during practical lessons (18 lessons);
- 2) three thematic tests;
- 3) individual tasks.

#### 2. Criteria for scoring points

##### 2.1. Responses during practical lessons:

- student has a thorough knowledge and understanding of the content and a high level of competence in the processes and skills - 1 point;
- student has achieved a basic level of competence in the processes and skills - 0.5 points;
- student has no elementary knowledge and understanding of the content - 0 points.

##### 2.2. Thematic test

- complete solutions (no less than 90% of the required information) - 12 points;
- solutions with minor inaccuracies (no less than 75% of the required information) - 9-11 points;
- mistakes in an solutions (no less than 50% of the required information) - 6-8 points;
- no solutions - 0 points.

##### 2.3. Individual tasks — 14 балів.

- complete solutions - 14 points;
- solutions with minor inaccuracies - 10-13 points;
- mistakes in an solutions - 7-9 points;
- no solutions - 0 points.

Each week of delay of the submission of the individual tasks to examination are provided with penalty (-5) points.

3. The requirement to the first attestation is to receive at least 11 points. The requirement to the second attestation is to receive at least 18 points.

4. The allowance requirement to passing the exam is the individual tasks, that must be credited, and no less than 36 points of starting score.

5. At the exam each student have two theoretical questions (8 points each) and three practical ones (8 points each). Total exam score is 40 points.

The criterion for evaluating theoretical questions:

- full answer (not less than 90% of the required information) - 8 points.
- sufficiently complete answer (not less than 75% of the required information, or minor inaccuracies) - 6-7 points.
- incomplete answer (not less than 60% of the required information and some mistakes) - 4-5 points.
- wrong answer - 0 points.

Criteria for evaluating practical tasks:

- complete solution of the task - 8 points.
- solution of the task with minor inaccuracies - 6-7 points.
- the task is solved with some mistakes - 4-5 points.
- the task is not solved - 0 points.

If the exam score is less than 24 points, then the student gets an "unsatisfactory" rating.

6. The sum of the starting score and the exam score (personal score) is transferred into the grade of credit module according to the table:

<b>PERSONAL SCORE</b>	<b>GRADE</b>
95-100	Excellent
85-94	Very Good
75-84	Good
65-74	Satisfactory
60-64	Sufficiently
Personal score < 60 or the exam score is less than 24 points	Unsatisfactory

Personal score < 36 or the individual tasks is not credited	Not allowed to pass exam
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